Übungsaufgaben Vorkurs Mathematik
Blatt 8
3. Aufgaben zum Thema trigonometrische Funktionen

44. Leiten Sie folgende Gleichungen her:

a) $\sinh(2x) = 2 \cdot \sinh x \cdot \cosh x$ b) $\operatorname{arsinh}\left[\frac{1}{1+x}\right] = \ln\left(1+\sqrt{x^2+2x+2}\right) - \ln(x+1)$

45. Lösen Sie die folgenden Gleichungen:

a) $4\sin x \cos x - 4\cos 2x = 2$ b) $\sin^2 x - \sqrt{1-\cos^2 x} = 0$

c) $\ln\cos x - \ln\sin x = 0$ d) $e^{\cos x} - e^{\sin 2x} = 0$

e) $\cos^2 x + 5\sin^2 x - 4\sin x = 0$ f) $\sin x \cdot \sin 2x = 0$

46. Wandeln Sie die folgenden komplexen Zahlen in Exponentialform um.

a) $5-5j$ b) $4-8j$ c) $15-13j$ d) $2+6j$ e) $-4+6j$ f) $-3-7j$

47. Geben Sie $z = 2{,}5 \cdot e^{j0{,}76}$ in der Form $a + jb$ an.

48. Berechnen Sie $e^{j146°} \cdot e^{-j82°}$ und schreiben Sie das Ergebnis in der Form $a + jb$.

49. Berechnen Sie:

a) $(1+j)^8$ b) $\left(1+j\sqrt{3}\right)^4$ c) $e^{j60°} \div e^{-j70°}$ d) $2e^{j30°} \cdot 4e^{j150°}$

e) $\dfrac{4\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)}{2\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)}$ f) $\sqrt{3}\left(\cos 50° + j\sin 50°\right) \cdot \sqrt{12}\left(\cos 10° - j\sin 10°\right)$

Stellen Sie alle komplexen Zahlen aus der Aufgabe graphisch dar.

50. Berechnen Sie x:

a) $\frac{5+x}{3-x} - \frac{8-3x}{x} = \frac{2x}{x-2}$ b) $\frac{(a-x)^2 + (x-b)^2}{(a-x)^2 - (x-b)^2} = \frac{a^2+b^2}{a^2-b^2}$ $a \neq b$

c) $4\sqrt[3]{6x-1} + 5 = 0$ d) $5\sqrt{2x+7} = \sqrt{50x+175}$ e) $\frac{1}{4}\sqrt{3x-5} + \sqrt{5x+2-3\sqrt{3x-5}} = 6$

f) $100^{x-1} = 10^{x^2-1}$ g) $3^{2x-1} + 9^{x-1} - 9^{x-2} = 27$ h) $6^{2x+4} = 3^{3x} \cdot 2^{x+8}$

i) $10^{2x} - 101 \cdot 10^x + 100 = 0$ j) $2^{\frac{3}{\sqrt{x}}} - 2^{\frac{2}{\sqrt{x}}-1} + 2^{\frac{1}{\sqrt{x}}} - 2 = 0$

k) $\lg x + \lg(x+1) - \lg(x-2) = 1$ l) $\log_{x-1}\left(x^2 - 5x + 2\right) = 2$

m) $x^{\lg x} - 21x^{-\lg x} - 8 = 0$ n) $\lg\left(3^{\sqrt{\frac{x(x-4)}{x-3}}} + 1\right) = 1$

Ergänzung zum Thema von Aufg. 49:

49 g) h) Benutze $\tan\frac{\pi}{8} = \sqrt{2} - 1$, um $\sin\frac{\pi}{8}$ sowie $\cos\frac{\pi}{8}$ ohne Taschenrechner exakt zu berechnen (also nicht als bloße Näherung). Benutze dann die Eulersche Formel, um z^4 zu berechnen.

g) $z = \frac{1}{\sqrt{4-2\sqrt{2}}} + \frac{j}{2}\sqrt{2-\sqrt{2}}$

h) $z = -\frac{1}{2}\sqrt{2-\sqrt{2}} + \frac{1}{2}\frac{j}{\sqrt{1-\frac{1}{2}\sqrt{2}}}$

$\rightarrow$ 44) a) Additionstheorem

$$\sinh(x+x) = \sinh x \cosh x + \cosh x \sinh x$$

$$\Rightarrow \quad \sinh(2x) = 2 \sinh x \cosh x$$

b) $$\operatorname{arsinh} z = \ln\left(z + \sqrt{z^2+1}\right)$$

$$z = \frac{1}{1+x}$$

$$\Rightarrow \operatorname{arsinh}\left(\frac{1}{1+x}\right) = \ln\left(\frac{1}{1+x} + \sqrt{\left(\frac{1}{1+x}\right)^2 + 1}\right)$$

$$= \ln\left(\frac{1}{1+x} + \sqrt{\frac{1+(1+x)^2}{(1+x)^2}}\right)$$

$$= \ln\left(\frac{1}{1+x} + \frac{1}{1+x}\sqrt{x^2+2x+2}\right)$$

$$= \ln\left(\frac{1+\sqrt{x^2+2x+2}}{1+x}\right)$$

$$= \ln\left(1+\sqrt{x^2+2x+2}\right) - \ln(1+x)$$

44b)

$$\sinh y = \frac{e^{y} - e^{-y}}{2} = z$$

$$\cancel{e^{2y} - 2z e^{y} - 1 = 0}$$

$$e^{2y} - 2z e^{y} - 1 = 0$$

$$e^{y} = z \pm \sqrt{z^2 + 1}$$

$$y = \ln\left(z + \sqrt{z^2+1}\right) = \operatorname{arsinh} z$$

$$\operatorname{Arsinh}\left(\frac{1}{1+x}\right) =$$

$$\operatorname{arsinh} z = \ln\left(z + z\sqrt{1+\frac{1}{z^2}}\right)$$

$$\operatorname{Arsinh}\left(\frac{1}{1+x}\right) = \ln\left(1 + \sqrt{x^2+2x+2}\right)$$

$$- \ln(x+1)$$

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→ 45)

a) $2\sin x\cos x - 2\cos 2x = 1$

$$\sin 2x - 2\cos 2x = 1$$

$$\frac{2\tan x}{1+\tan^2 x} - 2\cdot\frac{1-\tan^2 x}{1+\tan^2 x} = 1$$

$$\frac{2\tan x - 2 + 2\tan^2 x}{1+\tan^2 x} = 1$$

$$2\tan x - 2 + 2\tan^2 x = 1 + \tan^2 x$$

$$\tan^2 x + 2\tan x - 3 = 0$$

$$\tan x_{1,2} = -1 \pm \sqrt{4} = -1 \pm 2$$

$\tan x_1 = -3 \qquad x_1 = -71{,}56^\circ ; 108{,}44^\circ ; 288{,}44^\circ$

$\tan x_2 = 1 \qquad x_2 = 45^\circ ; 225^\circ$

Probe: alle 4 Lösungen

oder $\quad 2\sin x\cos x - 2\cos 2x = 1 \qquad \cos 2x = 1 - 2\sin^2 x$

$2\sin x\sqrt{1-\sin^2 x} - 2(1-2\sin^2 x) = 1 \qquad \cos x = \sqrt{1-\sin^2 x}$

$$2\sin x\sqrt{1-\sin^2 x} = 3 - 4\sin^2 x$$

$$4\sin^2 x(1-\sin^2 x) = 9 + 16\sin^4 x - 24\sin^2 x$$

$$0 = 20\sin^4 x - 28\sin^2 x + 9$$

$t = \sin^2 x \qquad 0 = 20t^2 - 28t + 9$

$$0 = t^2 - \frac{7}{5}t + \frac{9}{20}$$

$$t_{1,2} = \frac{7}{10} \pm \sqrt{\frac{49}{100} - \frac{9}{20}} = \frac{7}{10} \pm \sqrt{\frac{4}{100}} = \frac{7}{10} \pm \frac{2}{10}$$

$t_1 = \frac{1}{2} = \sin^2 x_1 \qquad \sin x_1 = \pm\frac{1}{\sqrt{2}} \qquad x_1 = 45^\circ ; \cancel{135^\circ} ; 225^\circ ; \cancel{315^\circ}$

$t_2 = \frac{9}{10} = \sin^2 x_2 \qquad \sin x_2 = \pm\frac{3}{\sqrt{10}} \qquad x_2 = \cancel{71{,}56^\circ} ; 108{,}44^\circ ; \cancel{251{,}56^\circ} ; 288{,}44^\circ$

Probe: $45^\circ, \cancel{315^\circ}$ 225°, $108{,}44^\circ, 288{,}44^\circ$

45a

$$\boxed{4\sin x\cos x - 4\cos(2x) = 2} \quad ; \; G = [-\tfrac{\pi}{2}, \tfrac{\pi}{2})$$

$|:2$ alle drei Terme sind π-periodisch

$$\Leftrightarrow \sin(2x) = 2\cos(2x) + 1 \quad |\text{Quad} \qquad (2)$$
$$\Rightarrow \sin^2(2x) = 4\cos^2(2x) + 1 + 4\cos(2x)$$
$$\Leftrightarrow 1 - \cos^2(2x) = 4\cos^2(2x) + 1 + 4\cos(2x)$$
$$\Leftrightarrow 0 = 5\cos^2(2x) + 4\cos(2x) \quad |:5 \,|\, \cos \text{ auskla.}$$
$$\Leftrightarrow 0 = \cos(2x)\left(\cos(2x) + \frac{4}{5}\right)$$

1\. Fall $\cos(2x) = 0 \Rightarrow$

$\sin(2x) = \underset{(-)}{+} 1 \Leftrightarrow 2x = \frac{\pi}{2} \Leftrightarrow x = \frac{\pi}{4}$ (keine weiteren Werte in G

kein "–" wegen (2), Probe durch (2) (sin, cos einsetzen!)

2\. Fall $\cos(2x) = -\frac{4}{5} \Rightarrow$

$\sin(2x) = \underset{-}{(+)}\sqrt{1-\cos^2(2x)} = -\frac{3}{5}$

kein "+" wegen (2), Probe durch (2) (sin, cos einsetzen!)

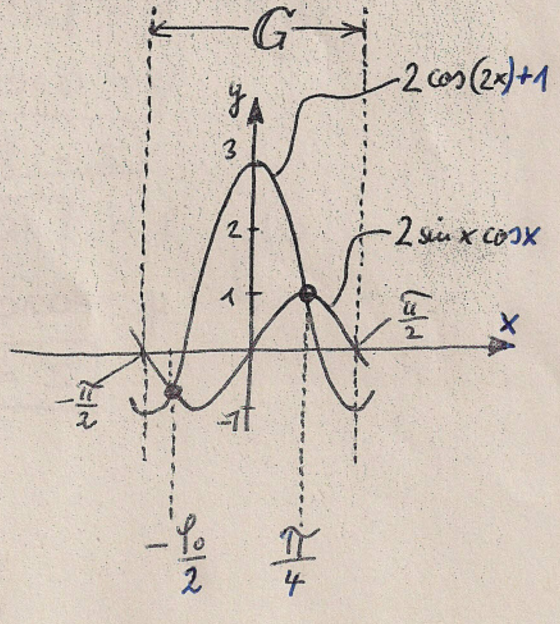
$\Leftrightarrow \underbrace{2x} = -\varphi_0 \; ; \; \varphi_0 := \arccos\left(-\frac{4}{5}\right)$

liegt im III. Quadranten, da φ_0 im II.
Muß so sein, da sin, cos beide < 0.

$\Leftrightarrow x = -\frac{\varphi_0}{2}$ (keine weiteren Werte in G)

$$\mathbb{L} = \left\{-\frac{\varphi_0}{2}, \frac{\pi}{4}\right\} \text{ im Grundbereich } G \; ; \; \varphi_0 := \arccos\left(-\frac{4}{5}\right)$$

Die Gleichung ist π-periodisch:
Wenn sie durch x gelöst wird,
so auch durch $x \pm \pi$.
Die komplette Lösungsmenge in
ganz $\mathbb{R}$ erhalte ich, indem ich
zu jeder Lösung x die weiteren
Lösungen $x \pm \pi$ hinzufüge und
zu diesen wieder solche u.s.w.



→ 45 b) $\sin^2 x - \sin x = 0 = \sin x(\sin x - 1)$

$G = [0, \pi]$, weil $\sin^2$, $\cos^2$ beide π-periodisch

$\Rightarrow \sin x = 0$ oder $\sin x = 1$

$\Rightarrow x = 0°; 180°; 90°;$ Probe ✓ und $\frac{3\pi}{2}$! falls $G = [0, 2\pi)$ definiert wird

c) $0 = \ln\left(\frac{\cos x}{\sin x}\right) \Rightarrow \frac{\cos x}{\sin x} = e^0 = 1$

$\Rightarrow \cot x = 1 \qquad x = 45°$

d) $e^{\cos x} = e^{\sin 2x} \qquad \cos x = \sin 2x = 2\sin x\cos x$

$1 = 2\sin x \qquad \sin x = \frac{1}{2} \qquad x = 30°; 150°$

falls $\cos x \neq 0$ sonst $x = \frac{\pi}{2}; \frac{3\pi}{2}$

e) $\cos^2 x + 5\sin^2 x - 4\sin x = 0$

$1 - \sin^2 x + 5\sin^2 x - 4\sin x = 0$

$4\sin^2 x - 4\sin x + 1 = 0$

$\sin^2 x - \sin x + \frac{1}{4} = 0$

$\sin x = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{4}} = \frac{1}{2} \qquad x = 30°, 150°$

f) $\sin x = 0$ oder $\sin 2x = 0$

$x = 0°, 180° \qquad 2x = 0°, 180°$

$x = 0°, 90°, 180°$

→ 46) a) $r = \sqrt{5^2 + 5^2} = \sqrt{50}$

$\varphi = \arctan\left(\frac{-5}{5}\right) + 360° = 315°$

$\boxed{z = \sqrt{50}\, e^{j\,315°}}$

b) $r = \sqrt{4^2 + 8^2} = \sqrt{80}$

$\varphi = \arctan\left(\frac{-8}{4}\right) + 360° = 296{,}57°$

$\boxed{z = \sqrt{80}\, e^{j\,296{,}57°}}$

c) $r = \sqrt{15^2 + 13^2} = \sqrt{394} = 19{,}85$

$\varphi = \arctan\left(-\frac{13}{15}\right) + 360° = 319{,}09°$

$\boxed{z = 19{,}85\, e^{j\,319{,}09°}}$

(20) → 46) d) $r = \sqrt{2^2+6^2} = \sqrt{40}$

$\varphi = \arctan\left(\frac{6}{3}\right) = 71{,}57^0$

$\boxed{z = \sqrt{40}\ e^{j\,71{,}57^0}}$

e) $r = \sqrt{4^2+6^2} = \sqrt{52}$

$\varphi = \arctan\left(\frac{6}{-4}\right) + 180^0 = 123{,}69^0$

$\boxed{z = \sqrt{52}\ e^{j\,123{,}69^0}}$

f) $r = \sqrt{3^2+7^2} = \sqrt{58}$

$\varphi = \arctan\left(\frac{-7}{-3}\right) + 180^0 = 246{,}8^0$

$\boxed{z = \sqrt{58}\ e^{j\,246{,}8^0}}$

→ 47) $2{,}5e^{j\,0{,}76} = 2{,}5(\cos 0{,}76 + j \sin 0{,}76)$

$= 2{,}5(0{,}7248 + j\ 0{,}6889)$

$= \boxed{1{,}812 + 1{,}722j}$

→ 48) $= e^{j(146^0 - 82^0)} = e^{j\,64^0} = \cos 64^0 + j \sin 64^0$

$= \boxed{0{,}4384 + 0{,}899j}$

→ 49) a) $1+j = \sqrt{2}\ e^{j\,45^0}$ $\qquad r = \sqrt{1^2+1^2} \quad \tan\varphi = 1$

$(1+j)^8 = (\sqrt{2}\ e^{j45^0})^8$

$= \sqrt{2}^8\ e^{j\cdot 8\cdot 45^0} = 16\ e^{j\,360^0}$

$= 16(\cos 360^0 + j \sin 360^0) = \boxed{16}$

b) $1 + j\sqrt{3} = 2e^{j60^0}$ $\qquad r = \sqrt{1^2+\sqrt{3}^2}$

$(1+j\sqrt{3})^4 = 2^4 \cdot e^{j\,4\cdot 60^0}$ $\qquad \tan\varphi = \sqrt{3}$

$= 16(\cos 240^0 + j \sin 240^0)$

$= \boxed{-8 - 8\sqrt{3}j}$

c) $e^{j\,60^0} \cdot e^{-j\,70^0} = \cancel{(\cos 60^0 + j\sin 60^0) \cdot (\cos 70^0 - j \sin 70^0)}$

$= \boxed{e^{j\,130^0}}$

$= \cancel{\frac{1}{2} + j\frac{1}{2}\sqrt{3} + 0{,}342 - 0{,}94j}$

$= \boxed{-0{,}6428 + 0{,}7660j}$ $\qquad = \cancel{0{,}842 + 0{,}074j}$

49d) ... $\Rightarrow = 8 \cdot e^{j180°} = \underline{\underline{-8}}$, weil $e^{j\pi} = -1$

e) ... $\Rightarrow = 2 \cdot e^{j\left(\frac{\pi}{4}-\frac{\pi}{2}\right)} = 2 \cdot e^{-j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right) = \underline{\underline{\sqrt{2} - j\sqrt{2}}}$

im 4. Quadranten

$\cos\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$
$\sin\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$

f) ... $\Rightarrow = \sqrt{3}\, e^{j50°} \cdot \sqrt{12}\, e^{-j10°}$

$= 6\, e^{j40°} = 6\left(\cos 40° + j\sin 40°\right)$

$= 6\left(0{,}7660 + j \cdot 0{,}6428\right) = \underline{\underline{4{,}5963 + j \cdot 3{,}856}}$

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g)h) allg.:

$\tan\frac{\pi}{8} = \sqrt{2} - 1$

$\sin\frac{\pi}{8} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\cos\frac{\pi}{8} = \frac{\sqrt{2}}{2\sqrt{2-\sqrt{2}}}$

(rechne nach!)

$\sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}}$ (rechne nac

$\cos x = \frac{\sin x}{\tan x}$ (warum?)

g) ... $z = a + jb$

ist wegen rechts

$= e^{j\frac{\pi}{8}}$

$\Rightarrow z^4 = e^{j\frac{\pi}{2}}$

$\Rightarrow \underline{\underline{z^4 = j}}$

$\tan\varphi = \frac{b}{a}$

$\tan\varphi = \frac{1}{2}\sqrt{2-\sqrt{2}} \cdot \sqrt{4-2\sqrt{2}} = \frac{\sqrt{2}}{2}(2-\sqrt{2})$

$= \sqrt{2} - 1 \Rightarrow \underline{\varphi = \frac{\pi}{8}}$

$r^2 = a^2 + b^2 = \left(\frac{1}{2}\sqrt{2-\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{4-2\sqrt{2}}}\right)^2$

$r^2 = \left(\frac{1}{2}\,\frac{2-\sqrt{2}}{\sqrt{2-\sqrt{2}}}\right)^2 + \left(\frac{1}{2}\,\frac{\sqrt{2}}{\sqrt{2-\sqrt{2}}}\right)^2 = \frac{\left(1 - \frac{1}{2}\sqrt{2}\right)^2 + \left(\frac{1}{2}\sqrt{2}\right)^2}{2 - \sqrt{2}}$

$r^2 = \frac{1 + \frac{1}{2} - \sqrt{2} + \frac{1}{2}}{2 - \sqrt{2}} = 1 \Rightarrow \underline{r = 1}$

h) ... so ähnlich mit dem Ergebnis: $\underline{z^4 = j}$

21 → 50) b) $\Rightarrow \dfrac{a^2+x^2-2ax + x^2+b^2-2bx}{a^2+x^2-2ax - x^2-b^2+2bx} = \dfrac{a^2+b^2}{a^2-b^2}$

$\Rightarrow (a^2+2x^2+b^2-2ax-2bx)(a^2-b^2) = (a^2+b^2)(a^2-b^2-2ax+2bx)$

$\Rightarrow \cancel{a^4}+2a^2x^2+\cancel{a^2b^2}-\cancel{2a^3x}-2a^2bx-\cancel{a^2b^2}-2b^2x^2-\cancel{b^4}+2ab^2x$
$+\cancel{2b^3x} = \cancel{a^4}-\cancel{a^2b^2}-\cancel{2a^3x}+2a^2bx+\cancel{a^2b^2}-\cancel{b^4}-2ab^2x+\cancel{2b^3x}$

$\Rightarrow -2b^2x^2+2a^2x^2-2a^2bx+2ab^2x = 2a^2bx-2ab^2x \quad | : x$

$\boxed{x_1 = 0}$

$-2b^2x+2a^2x-2a^2b+2ab^2 = 2a^2b-2ab^2$

$\Rightarrow \quad -2b^2x+2a^2x = 4a^2b-4ab^2 = 4ab(a-b)$

Probe ✓ $\quad x_2 = \dfrac{4ab(a-b)}{2(a^2-b^2)} = \underline{\underline{\dfrac{2ab}{a+b}}}$

c) $4\cdot\sqrt[3]{6x-1} = -5 \qquad \sqrt[3]{6x-1} = -\dfrac{5}{4}$

$6x-1 = \left(-\dfrac{5}{4}\right)^3 = -\dfrac{125}{64}$

$6x = 1-\dfrac{125}{64} = -\dfrac{61}{64} \qquad x = -\dfrac{61}{384} = -0{,}1588$

d) $25(2x+7) = 50x+175$

$50x+175 = 50x+175 \quad \Rightarrow x$ beliebig

e) $\dfrac{1}{16}(3x-5)+36-\cancel{3\sqrt{3x-5}} = 5x+2-\cancel{3\sqrt{3x-5}}$

$\dfrac{1}{16}(3x-5)+36 = 5x+2$

$3x-5+576 = 80x+32$

$0 = 77x-539$

$x = \dfrac{539}{77} = 7 \qquad$ Probe ✓

f) $(x-1)\lg 100 = (x^2-1)\lg 10$

$2(x-1) = x^2-1$

$0 = x^2-2x+1 = (x-1)^2 \qquad x = 1$

Probe ✓

g) $\frac{1}{3} 3^{2x} + \frac{1}{9} 3^{2x} - \frac{1}{3^4} 3^{2x} = 27$

$3^{2x}\left(\frac{1}{3} + \frac{1}{9} - \frac{1}{81}\right) = 27$

$3^{2x}\left(\frac{27+9-1}{81}\right) = 27$

$3^{2x} \cdot \frac{35}{81} = 27 \qquad 3^{2x} = \frac{27 \cdot 81}{35}$

$2x \lg 3 = \lg\left(\frac{27 \cdot 81}{35}\right)$

$x = \frac{\lg\left(\frac{27 \cdot 81}{35}\right)}{2 \lg 3} = 1{,}882$ Probe ✓

h) $(2x+4) \lg 6 = 3x \lg 3 + (x+8) \lg 2$

$2x \lg 6 + 4 \lg 6 = 3x \lg 3 + x \lg 2 + 8 \lg 2$

$x(2 \lg 6 - 3 \lg 3 - \lg 2) = 8 \lg 2 - 4 \lg 6$

$x = \frac{8 \lg 2 - 4 \lg 6}{2 \lg 6 - 3 \lg 3 - \lg 2} = 4$ Probe ✓

i) $(10^x)^2 - 101 \cdot 10^x + 100 = 0 \qquad t = 10^x$

$t^2 - 101\, t + 100 = 0$

$t_{1,2} = \frac{101}{2} \pm \sqrt{\frac{101^2}{4} - 100} = 50{,}5 \pm 49{,}5$

$t_1 = 100 = 10^{x_1} \qquad x_1 = 2$

$t_2 = 1 = 10^{x_2} \qquad x_2 = 0$ Probe ✓

j) $\left(2^{\frac{1}{\sqrt{3}}}\right)^3 - \frac{1}{2}\left(2^{\frac{1}{\sqrt{x}}}\right)^2 + 2^{\frac{1}{\sqrt{x}}} - 2 = 0 \qquad t = 2^{\frac{1}{\sqrt{3}}}$ f

$t^3 - \frac{1}{2} t^2 + t - 2 = 0$

$t = 0 \qquad f(t) = -2$

$t = 1 \qquad f(t) = -\frac{1}{2}$

$t = 2 \qquad f(t) = 6$

Die Funktion ist monoton steigend

→ nur eine Nullstelle

(22) 50j)

$t = 2{,}5 \qquad f(t) = 13$

$t = 1{,}5 \qquad f(t) = 1{,}75$

$t = 1{,}25 \qquad f(t) = 0{,}422$

$t = 1{,}2 \qquad f(t) = 0{,}208$

$t = 1{,}1 \qquad f(t) = -0{,}174$

$t = 1{,}15 \qquad f(t) = 0{,}0096$

$\vdots$

$t = 1{,}1475 \qquad f(t) = 0{,}0000996$

$t = 2^{\frac{1}{\sqrt{x}}} \qquad \lg t = \frac{1}{\sqrt{x}} \lg 2$

$\sqrt{x} = \frac{\lg 2}{\lg t} \qquad x = \left(\frac{\lg 2}{\lg t}\right)^2 = 25{,}38$

Probe ✓

k) $\lg\left(\frac{x(x+1)}{x-2}\right) = \lg 10$

$\frac{x(x+1)}{x-2} = 10$

$x(x+1) = 10(x-2) \qquad x^2 + x = 10x - 20$

$x^2 - 9x + 20 = 0 \qquad x_{1,2} = 4{,}5 \pm \sqrt{0{,}25} = 4{,}5 \pm 0{,}5$

$x_1 = 5 \qquad x_2 = 4$ Probe ✓

l) $(x-1)^2 = x^2 - 5x + 2$

$\cancel{x^2} - 2x + 1 = \cancel{x^2} - 5x + 2$

$3x = 1 \qquad x = \frac{1}{3}$

$x - 1 = \frac{1}{3} - 1 = -\frac{2}{3} \Rightarrow \lg$ nicht def.

$\Rightarrow$ keine Lösung

m) $t = x^{\lg x}$

$t - \frac{21}{t} - 8 = 0$

50 m) $t^2 - 21 - 8t = 0$

$t_{1,2} = 4 \pm \sqrt{16+21} = 4 \pm \sqrt{37}$

$t_1 = 10{,}083 = x_1^{\lg x_1}$

$(\lg x_1)^2 = \lg 10{,}083$

$\lg x_1 = \pm \sqrt{\lg 10{,}083} = \pm 1{,}0018$

$x_1 = 10{,}04 \qquad x_2 = 9{,}96 \cdot 10^{-2}$

$t_2 = -2{,}083 \Rightarrow (\lg x)^2 = \lg(-2{,}083)$

$\Rightarrow$ keine Lösung

Probe: x_1 ist Lösung

n) $3^{\sqrt{\frac{x(x-4)}{x-3}}} + 1 = 10 \qquad 3^{\sqrt{\frac{x(x-4)}{x-3}}} = 9$

$\sqrt{\frac{x(x-4)}{x-3}} \lg 3 = \lg 9 \qquad \sqrt{\frac{x(x-4)}{x-3}} = \frac{\lg 9}{\lg 3} = 2$

$\frac{x(x-4)}{x-3} = 4 \qquad x(x-4) = 4(x-3)$

$x^2 - 4x = 4x - 12 \qquad x^2 - 8x + 12 = 0$

$x_{1,2} = 4 \pm \sqrt{4} = 4 \pm 2$

$x_1 = 6 \qquad x_2 = 2$

Probe: ✓